

4-4 Powers of Monomials

Objective: To find powers of monomials.

Rules of Exponents	Examples
<p>Rule of Exponents for a Power of a Power</p> <p>For all positive integers m and n:</p> $(a^m)^n = a^{mn}$ <p>To find a power of a power, you multiply the exponents.</p>	$(2^3)^4 = 2^{3 \cdot 4}$ $= 2^{12}$
<p>Rule of Exponents for a Power of a Product</p> <p>For every positive integer m:</p> $(ab)^m = a^m b^m$ <p>To find a power of a product, you find the power of each factor and then multiply.</p>	$(-2x)^5 = (-2)^5(x)^5$ $= -32x^5$

CAUTION $(x^7)^6 = x^{7 \cdot 6} = x^{42}$ but $x^7 \cdot x^6 = x^{7+6} = x^{13}$

Example 1 Simplify: a. $(x^2)^4$ b. $(u^3)^5$

Solution Use the rule for a power of a power.

$$\begin{array}{ll} \text{a. } (x^2)^4 = x^{2 \cdot 4} & \text{b. } (u^3)^5 = u^{3 \cdot 5} \\ = x^8 & = u^{15} \end{array}$$

Simplify.

1. $(a^2)^3$ a^6
2. $(x^4)^3$ x^{12}
3. $(r^5)^3$ r^{15}
4. $(c^3)^3$ c^9
5. $(r^2)^3$ r^6
6. $(x^5)^2$ x^{10}
7. $(y^{10})^3$ y^{30}
8. $(a^7)^8$ a^{56}

Example 2 Simplify: a. $(2x)^4$ b. $(-6k)^3$

Solution Use the rule for a power of a product.

$$\begin{array}{ll} \text{a. } (2x)^4 = 2^4 \cdot x^4 & \text{b. } (-6k)^3 = (-6)^3 \cdot k^3 \\ = 16x^4 & = -216k^3 \end{array}$$

Simplify.

9. $(5a)^2$ $25a^2$
10. $(-6x)^2$ $36x^2$
11. $(-3r)^3$ $-27r^3$
12. $(-4c)^2$ $16c^2$
13. $(-5x)^3$ $-125x^3$
14. $(-4r)^3$ $-64r^3$
15. $(-2t)^4$ $16t^4$
16. $(6x)^3$ $216x^3$
17. $(5x)^4$ $625x^4$
18. $(7n)^2$ $49n^2$
19. $(\frac{1}{2}a)^2$ $\frac{1}{4}a^2$
20. $(-\frac{1}{3}a)^3$ $-\frac{1}{27}a^3$

4-4 Powers of Monomials (continued)

Example 3 Evaluate if $x = 3$: a. $2x^3$ b. $(2x)^3$ c. $2^3 \cdot x^3$

Solution

$$\begin{array}{lll} \text{a. } 2x^3 = 2(3)^3 & \text{b. } (2x)^3 = (2 \cdot 3)^3 & \text{c. } 2^3 \cdot x^3 = 2^3 \cdot 3^3 \\ = 2(27) & = 6^3 & = 8 \cdot 27 \\ = 54 & = 216 & = 216 \end{array}$$

Evaluate if $x = 2$ and $y = 4$.

21. a. $2x^3$ 16
22. a. $4y^2$ 64
23. a. x^2y^3 256
- b. $(2x)^3$ 64
- b. $(4y)^2$ 256
- b. x^2y^2 64
- c. $2^3 \cdot x^3$ 64
- c. $4^2 \cdot y^2$ 256
- c. $(xy)^2$ 64
24. a. xy^3 128
25. a. $3x^2$ 12
26. a. $5x^2$ 20
- b. $(xy)^3$ 512
- b. $(3x)^2$ 36
- b. $(5x)^2$ 100
- c. $x^3 \cdot y^3$ 512
- c. $3^2 \cdot x^2$ 36
- c. $5^2 \cdot x^2$ 100
27. a. xy^2 32
28. a. $2xy$ 16
29. a. $6x^2 \div x$ 12
- b. $(x^2y)^2$ 256
- b. $2x^2y$ 32
- b. $(6x)^2 \div x$ 72
- c. x^3y 32
- c. $2xy^2$ 64
- c. $6(x^2 \div x)$ 12

Example 4 Simplify $(-2x^2y^3)^4$.

Solution $(-2x^2y^3)^4 = (-2)^4(x^2)^4(y^3)^4$ { First use the rule for a power of a product
 $= 16x^8y^{12}$ and then use the rule for a power of a power.

Simplify.

30. $(3n^2)^3$ $27n^6$
31. $(6b^4)^2$ $36b^8$
32. $(\frac{1}{3}x^{10})^3$ $\frac{1}{27}x^{30}$
33. $(\frac{1}{2}x^2)^4$ $\frac{1}{16}x^8$
34. $(2ab^2)^3$ $8a^3b^6$
35. $(-3x^2y^3)^3$ $-27x^6y^9$
36. $(4x^3y^2)^3$ $64x^9y^6$
37. $(-2xy^2)^4$ $16x^4y^8$
38. $(5m^2n^4)^2$ $25m^4n^8$

Mixed Review Exercises

Simplify.

1. $(2a^2b)(3ab)(5ab^2)$ $30a^4b^4$
2. $(-xy^2)(2xy)(-3y)$ $6x^2y^4$
3. $(3x^2y^3)^4$ $81x^8y^{12}$
4. $(\frac{1}{3}t^2)(\frac{3}{4}t^3)$ $\frac{1}{4}t^5$
5. $5c - 2a - 3c + a$ $2c - a$
6. $(2x + 3y + 1) + (3x + 2y)$ $5x + 5y + 1$
7. $3 \cdot 5^2 + 3 \cdot 5$ 90
8. $-3^2 \cdot 4$ -36
9. $(3^3 + 5^2) \div 2^2$ 13
10. $7 \cdot 3^2 + 6 \cdot 3 + 2$ 83
11. $(\frac{5}{2}t^2)(\frac{1}{5}t^3)$ $\frac{1}{2}t^5$
12. $(15mn^2)(\frac{1}{3}m^2)(4n)$ $20m^3n^3$